Homework — Part 1: Review of the lesson

6 problems. Solve them in your notebook. Problem 5 is Paper 1 style (no calculator, exact answers). Problem 6 is Paper 2 style (calculator allowed).

Key formulas

Area between a curve and the y-axis: $A = \int_c^d x\,dy$ (write x in terms of y; limits are **y-values**)

About the x-axis: $V = \pi \int_a^b y^2\,dx$ About the y-axis: $V = \pi \int_c^d x^2\,dy$

Two curves about the x-axis (y_1 above y_2): $V = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx$

Give units^2 or units^3. Calculator answers: 3 significant figures.

Level 1 — Basic

1. The region is bounded by the curve $y = x^2 + 1$ (for $x \geq 0$), the y-axis and the lines $y = 2$ and $y = 10$.

(a) **Write down** x in terms of y. [1]

(b) **Find** the area of the region. [3]

2. The region under the curve $y = x^3$, for $0 \leq x \leq 1$, is rotated through 2π about the x-axis. **Find** the exact volume of the solid formed. [3]

Level 2 — Intermediate

3. The part of the curve $y = \dfrac{1}{x}$ between $y = 1$ and $y = 2$ is rotated through 2π about the y-axis. **Find** the exact volume of the solid formed. [4]

4. The curves $y = 2x$ and $y = x^2$ enclose a region.

(a) **Find** the x-coordinates of the points where the curves meet. [2]

(b) The region is rotated through 2π about the x-axis. **Find** the exact volume of the solid formed. [4]

Level 3 — Exam-style

5. Paper 1 — no calculator

The region R is bounded by the curve $y = \sin x$ and the x-axis, for $0 \leq x \leq \pi$.

(a) **Show that** $\sin^2 x = \dfrac{1 - \cos 2x}{2}$. [1]

(b) R is rotated through 2π about the x-axis. **Find** the exact volume of the solid formed. [5]

6. Paper 2 — calculator

The region R is bounded by the curve $y = \dfrac{1}{\sqrt{1+x^2}}$, the x-axis, the y-axis and the line $x = 2$.

(a) **Find** the area of R. **[2]**

(b) R is rotated through 2π about the x-axis. **Find** the volume of the solid formed. **[3]**

(c) The part of the curve with $x \geq 0$ and $\dfrac{1}{\sqrt{5}} \leq y \leq 1$ is rotated through 2π about the y-axis. **Find** the volume of the solid formed. **[4]**

Answers (short)

1	(a) $x = \sqrt{y-1}$ (b) $\frac{52}{3}$ units² (≈ 17.3)
2	$\frac{\pi}{7}$ units³
3	$\frac{\pi}{2}$ units³
4	(a) $x = 0$ and $x = 2$ (b) $\frac{64\pi}{15}$ units³
5	(b) $\frac{\pi^2}{2}$ units³
6	(a) 1.44 units² (b) 3.48 units³ (c) 2.15 units³