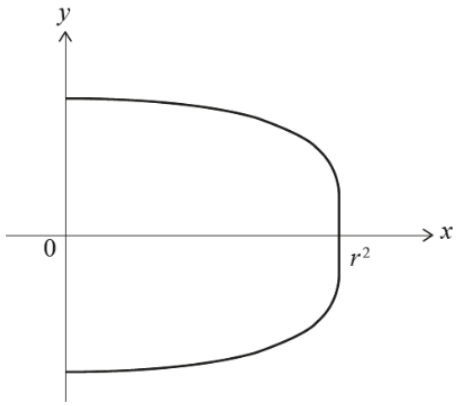
Homework — Part 2: Exam practice (IB Questionbank)

10 questions, 64 marks. Paper 1 = no calculator (exact answers). Paper 2 = calculator allowed. Show every step. Unless the question says otherwise, give calculator answers to 3 significant figures.

1. Paper 1 25M.1.AHL.TZ3.7 **[6]**

The curve $x^2 + y^4 = r^4$, where $0 \leq x \leq r^2$, is shown in the following diagram.



The region enclosed by the curve and the y-axis is rotated through 2π radians about the y-axis to form a solid of revolution.

Find an expression for the volume of the solid in the form $V = a\pi r^b$, where $a,\ b \in \mathbb{Q}^+$.

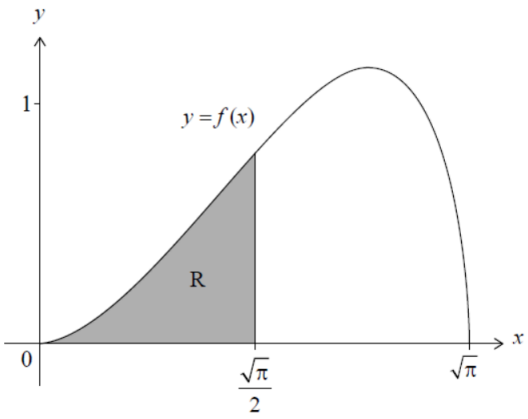
2. Paper 2 26M.2.AHL.TZ2.5 **[5]**

Part of the curve $y = \dfrac{20}{x+1}$ between point A $(1,\ 10)$ and point B $(4,\ 4)$ is rotated 360° about the y-axis to form a solid of revolution.

Find the volume of the solid formed.

3. Paper 1 24M.1.AHL.TZ1.6 **[6]**

The function f is defined as $f(x) = \sqrt{x \sin(x^2)}$, where $0 \leq x \leq \sqrt{\pi}$. Consider the shaded region R enclosed by the graph of f, the x-axis and the line $x = \frac{\sqrt{\pi}}{2}$, as shown in the following diagram.



The shaded region R is rotated by 2π radians about the x-axis to form a solid.

Show that the volume of the solid is $\frac{\pi\left(2-\sqrt{2}\right)}{4}$.

4. Paper 2 24M.2.AHL.TZ2.7 **[5]**

The curve $y = 4\ln(x-2)$ for $0 \leq y \leq 4$ is rotated $360°$ about the y-axis to form a solid of revolution.

Find the volume of the solid formed.

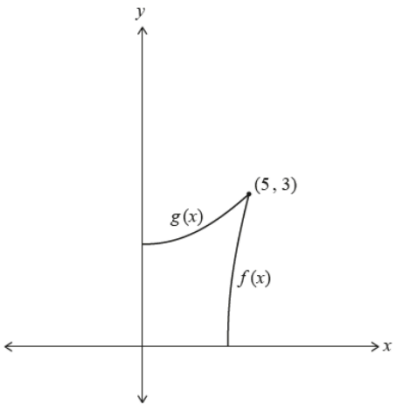
5. Paper 1 24N.1.AHL.TZ0.7 (part (b) only) **[4]**

Consider the function $f(x) = \sec\left(x - \frac{\pi}{4}\right)$, for $0 \leq x \leq \frac{\pi}{2}$. The region bounded by the graph of $y = f(x)$, the x-axis and the lines $x = 0$ and $x = \frac{\pi}{2}$ is rotated 2π radians about the x-axis.

Find the volume of revolution generated.

6. Paper 2 25N.2.AHL.TZ1.8 **[6]**

The following diagram shows the graphs of two functions, f and g, where $f(x) = \sqrt{x^2 - 16}$ for $4 \leq x \leq 5$, and $g(x) = 2 + \frac{x^2}{25}$ for $0 \leq x \leq 5$.

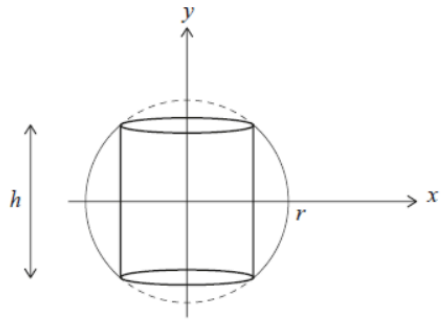


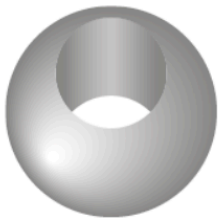
The graphs of f and g intersect at the point $(5, 3)$. The region bounded by the graphs of f, g and both axes is rotated $360°$ about the y-axis to form a solid of revolution.

Find the volume of the solid formed.

7. Paper 1 23M.1.AHL.TZ1.9 **[7]**

The function f is defined by $f(y) = \sqrt{r^2 - y^2}$ for $-r \leq y \leq r$. The region enclosed by the graph of $x = f(y)$ and the y-axis is rotated by $360°$ about the y-axis to form a solid sphere. The sphere is drilled through along the y-axis, creating a cylindrical hole. The resulting spherical ring has height h. This information is shown in the following diagrams. *(diagram not to scale)*



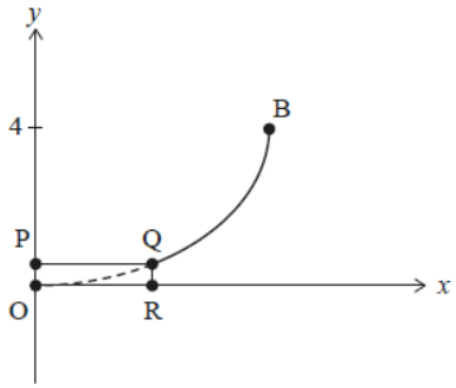


The spherical ring has a volume of π cubic units.

Find the value of h.

8. Paper 2 22M.2.AHL.TZ2.6 **[5]**

The following diagram shows the curve $\frac{x^2}{36} + \frac{(y-4)^2}{16} = 1$, where $h \leq y \leq 4$. *(diagram not to scale)*



The curve from point Q to point B is rotated $360°$ about the y-axis to form the interior surface of a bowl. The rectangle OPQR, of height h cm, is rotated $360°$ about the y-axis to form a solid base. The bowl is assumed to have negligible thickness.

Given that the interior volume of the bowl is to be $285\ \text{cm}^3$, **determine** the height of the base.

9. Paper 1 25M.1.AHL.TZ1.12 (parts (a)–(d)) **[14]**

Consider the family of functions $f_n(x) = \cos^n x$, where $x \in \mathbb{R}$ and $n \in \mathbb{N}$.

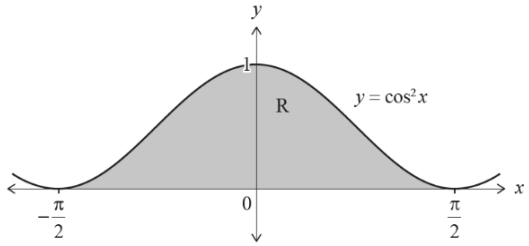
(a) By writing $\cos^n x$ as $\cos^{n-1} x \cos x$, **show that** **[4]**

$\int \cos^n x\, dx = \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x\, dx - (n-1)\int \cos^n x\, dx$ for $n > 1$.

(b) **Hence, show that** $\int f_n(x)\, dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int f_{n-2}(x)\, dx$ for $n > 1$. **[2]**

(c) **Hence, find** an expression for $\int \cos^4 x\, dx$, giving your answer in the form $p\cos^3 x \sin x + q \cos x \sin x + rx + c$ where $p,\ q,\ r \in \mathbb{Q}^+$. **[4]**

The region R is enclosed by the graph of $y = \cos^2 x$ and the x-axis where $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, as shown in the following diagram.

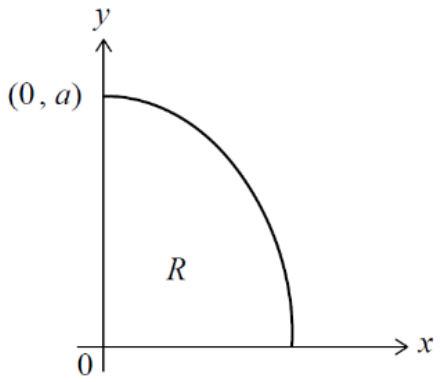


The region R is rotated by 2π radians around the x-axis to form a solid of revolution.

(d) **Find** the volume of the solid. **[4]**

10. Paper 2 26M.2.AHL.TZ3.6 **[6]**

The following graph shows the curve $y = \sqrt{9\left(1 - \frac{x^2}{4}\right)}$ for $0 \leq x \leq 2$. The curve intersects the y-axis at the point $(0,\ a)$. *(diagram not to scale)*



(a) **Write down** the value of a. **[1]**

The region R is bounded by the curve, the x-axis and the y-axis. R is rotated through $360°$ about the y-axis to form a solid.

(b) **Find** the volume of this solid. **[5]**

Questions © IBO. For use by students of this class only.