Homework — Part 1: Review of integration

6 problems. Solve them in your notebook. Show every step. **Paper 1** = no calculator, **Paper 2** = Casio allowed.

Choose the method: power / e^{kx} / $\frac{1}{x}$ / $\sin$, $\cos$ → standard integral; "function × its derivative" → substitution; "$x \times e^x$, $\sin x$, $\ln x$" → by parts; fraction with factors → partial fractions. Area $= \int_a^b (\text{top} - \text{bottom})\,dx$; volume about the x-axis $V = \pi \int_a^b y^2\,dx$. Do not forget $+C$.

Level 1 — Basic

1. Paper 1 **Find** $\displaystyle\int \left(6x^2 + 2e^x - \frac{1}{x}\right) dx.$ [3]

2. Paper 1 **Find** the area of the region enclosed by $y = x(4 - x)$ and the x-axis. [4]

Level 2 — Intermediate

3. Paper 1

(a) Use integration by parts to **find** $\displaystyle\int xe^{2x}\,dx.$ [3]

(b) **Hence** find the exact value of $\displaystyle\int_0^1 xe^{2x}\,dx.$ [2]

4. Paper 1

(a) **Write** $\dfrac{3x+1}{(x-1)(x+1)}$ in partial fractions. [2]

(b) **Hence** find $\displaystyle\int_2^3 \frac{3x+1}{(x-1)(x+1)}\,dx$, giving your answer in the form $\ln k$. [4]

Level 3 — Exam-style

5. Paper 1 Let $f(x) = \dfrac{x}{1+x^2}$, $x \geq 0$.

(a) Use the substitution $u = 1 + x^2$ to **find** $\displaystyle\int f(x)\,dx.$ [3]

(b) **Find** the exact area of the region bounded by the graph of f, the x-axis and the line $x = \sqrt{3}$. [2]

(c) The area between the graph of f, the x-axis and the line $x = k$ is $\frac{1}{2}\ln 2$. **Find** k. [2]

6. Paper 2 Let $g(x) = xe^{-x}$, $0 \leq x \leq 4$.

(a) **Find** the coordinates of the maximum point of the graph of g. [2]

(b) **Find** the area of the region between the graph of g and the x-axis. [2]

(c) This region is rotated through 2π about the x-axis. **Find** the volume of the solid. [2]

(d) The area under the graph of g from $x = 0$ to $x = a$ is 0.5. **Find** a. [2]

Answers (short)

1	$2x^3 + 2e^x - \ln\lvert x\rvert + C$
2	$\frac{32}{3}$
3	(a) $\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$ (b) $\frac{e^2+1}{4}$
4	(a) $\frac{2}{x-1} + \frac{1}{x+1}$ (b) $\ln\frac{16}{3}$
5	(a) $\frac{1}{2}\ln(1+x^2) + C$ (b) $\ln 2$ (c) $k = 1$
6	(a) $(1,\ 0.368)$ (b) 0.908 (c) 0.775 (d) $a = 1.68$