Homework — Part 1: Rates of change and improper integrals

6 problems. Solve them in your notebook. Show every step. **Paper 1** = no calculator, **Paper 2** = Casio allowed (radians!).

Total change $= \int_a^b (\text{rate})\,dt$ = area under the rate graph. End amount = start amount + net change. Units = rate units × time units.

Improper integral: $\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$ — always write the limit. $\int_1^\infty x^p\,dx$ exists only for $p < -1$.

Level 1 — Basic

1. Paper 1 Water flows into a pool at a rate of $R(t) = 12 - 2t$ m³ per hour, $0 \le t \le 6$. At $t = 0$ the pool contains 20 m³ of water.

(a) **Find** the volume of water that flows in during the 6 hours. [2]

(b) **Write down** the volume of water in the pool at $t = 6$. [1]

2. Paper 1

(a) **Evaluate** $\int_1^\infty \frac{2}{x^3}\,dx$. [3]

(b) **Show that** $\int_1^\infty \frac{2}{x}\,dx$ does not exist. [2]

Level 2 — Intermediate

3. Paper 2 A reservoir contains 1000 m³ of water at $t = 0$. Water flows in at $R_{in}(t) = 50 + 10\sin\left(\frac{\pi t}{6}\right)$ m³ per hour and flows out at a constant 45 m³ per hour, $0 \le t \le 12$.

(a) **Find** the amount of water in the reservoir at $t = 12$. [3]

(b) **Find** the time when the amount of water is greatest, and **find** this amount. [4]

4. Paper 1

(a) Use the substitution $u = x^2$ to **evaluate** $\int_0^\infty xe^{-x^2}\,dx$. [4]

(b) **Show that** $\int_1^\infty \frac{1}{x(x+1)}\,dx = \ln 2$. [5]

Level 3 — Exam-style

5. Paper 1 Let R be the region under the graph of $y = \frac{1}{x}$ for $x \geq 1$.

(a) **Show that** the area of R is not finite. **[2]**

(b) R is rotated through 2π about the x-axis. **Show that** the volume of the solid is π. **[3]**

6. Paper 2 A drug enters the blood at a rate of $R(t) = 20te^{-0.4t}$ mg per hour, where $t \geq 0$ is in hours.

(a) **Find** the maximum rate and the time when it occurs. **[2]**

(b) **Find** the amount of drug that enters the blood in the first 5 hours. **[2]**

(c) **Show that** the total amount that enters the blood, if the process continues forever, is 125 mg. **[4]**

Answers (short)

1	(a) 36 m³ (b) 56 m³
2	(a) 1 (b) $2 \ln b \to \infty$
3	(a) 1060 m³ (b) $t = 7$ h, 1071 m³
4	(a) $\frac{1}{2}$ (b) shown
5	(a) $\ln b \to \infty$ (b) shown
6	(a) 18.4 mg/h at $t = 2.5$ h (b) 74.2 mg (c) shown